# C & EE 141

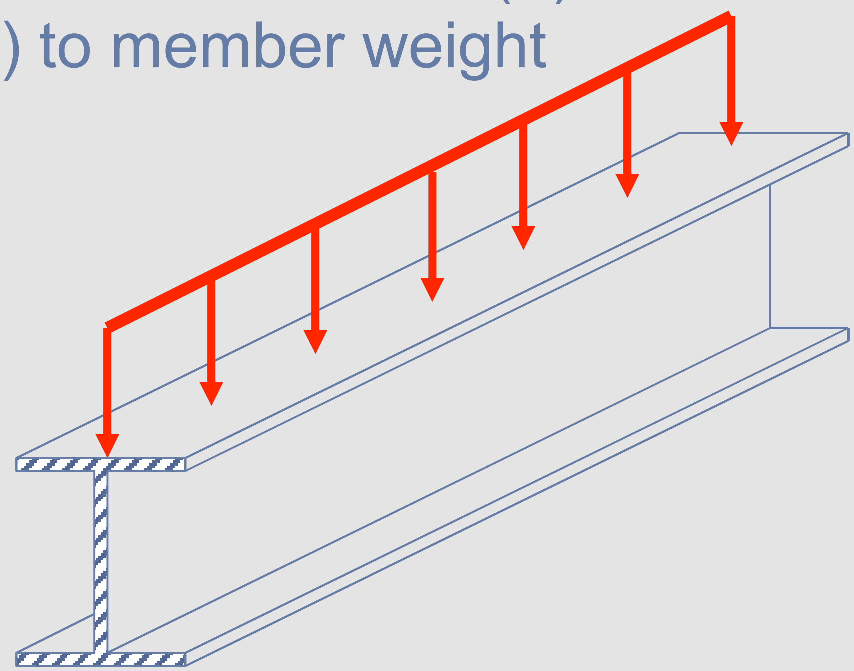
## Bending Members:
## Laterally Supported and Compact

# Flexural Members

- Structural members which support transverse loads, and are therefore subjected primarily to bending.
- For pure flexural members, axial load is negligible and ignored.
- If axial load is significant, it is a beam-column (we'll address that in later lectures).
- Also subjected to shear forces.
- Addressed in Part 3 of SCM and Section F of Spec

# Wide Flange Beams

- Focus on Wide Flange shapes
  - Most common beam shape due to economy.
    - High ratio of plastic section modulus (Z) and moment of inertia (I) to member weight
    - Easy to connect

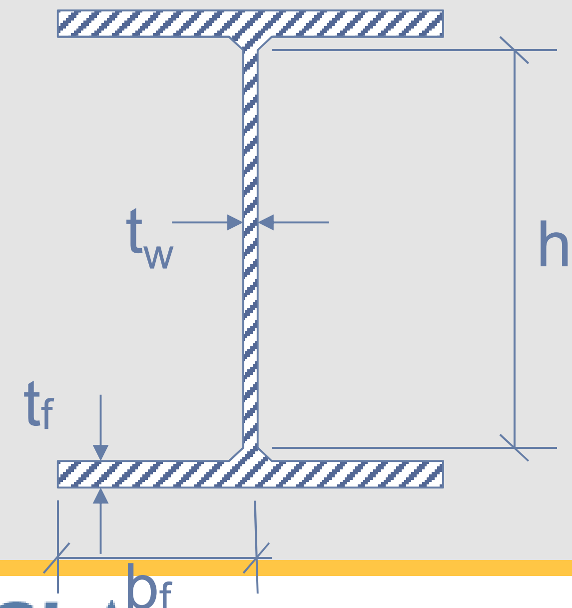


# Laterally Supported Beams

- Compression flange (usually top) is restrained against global buckling due to compression component of bending stresses.
  - Metal deck puddle-welded to top flange of beam.
  - Headed studs engaging a concrete deck or slab.
  - Other details that prevent lateral movement of the flanges.

# Compact Beams

- Similar to “non-slender” sections for compression
- Compact beams can develop yield stresses due to flexure without local buckling of any elements of the section.




$$h/t_w \leq 3.76\sqrt{E/F_y}$$

(Spec Table B4.1b #15)

$$b/t \leq 0.38\sqrt{E/F_y}$$

(Spec Table B4.1b #10)

Shapes with non-compact webs and/or flanges must consider local buckling limit states. (Much more complicated).

# Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

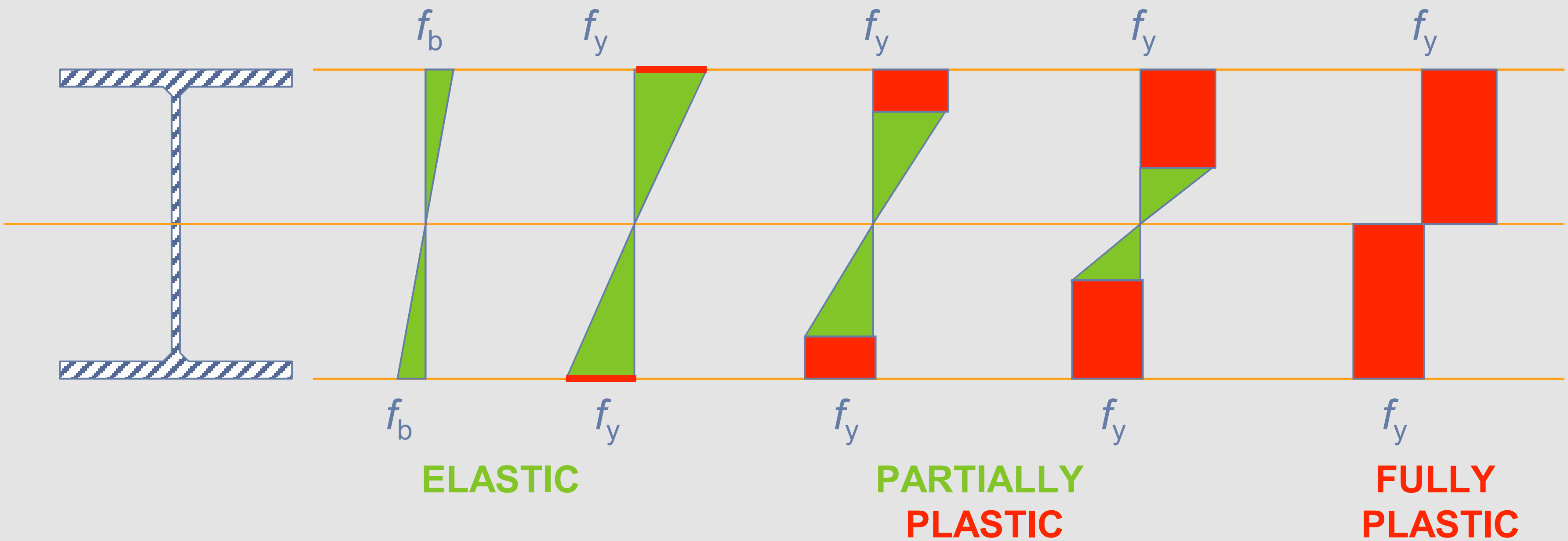
- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

# Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

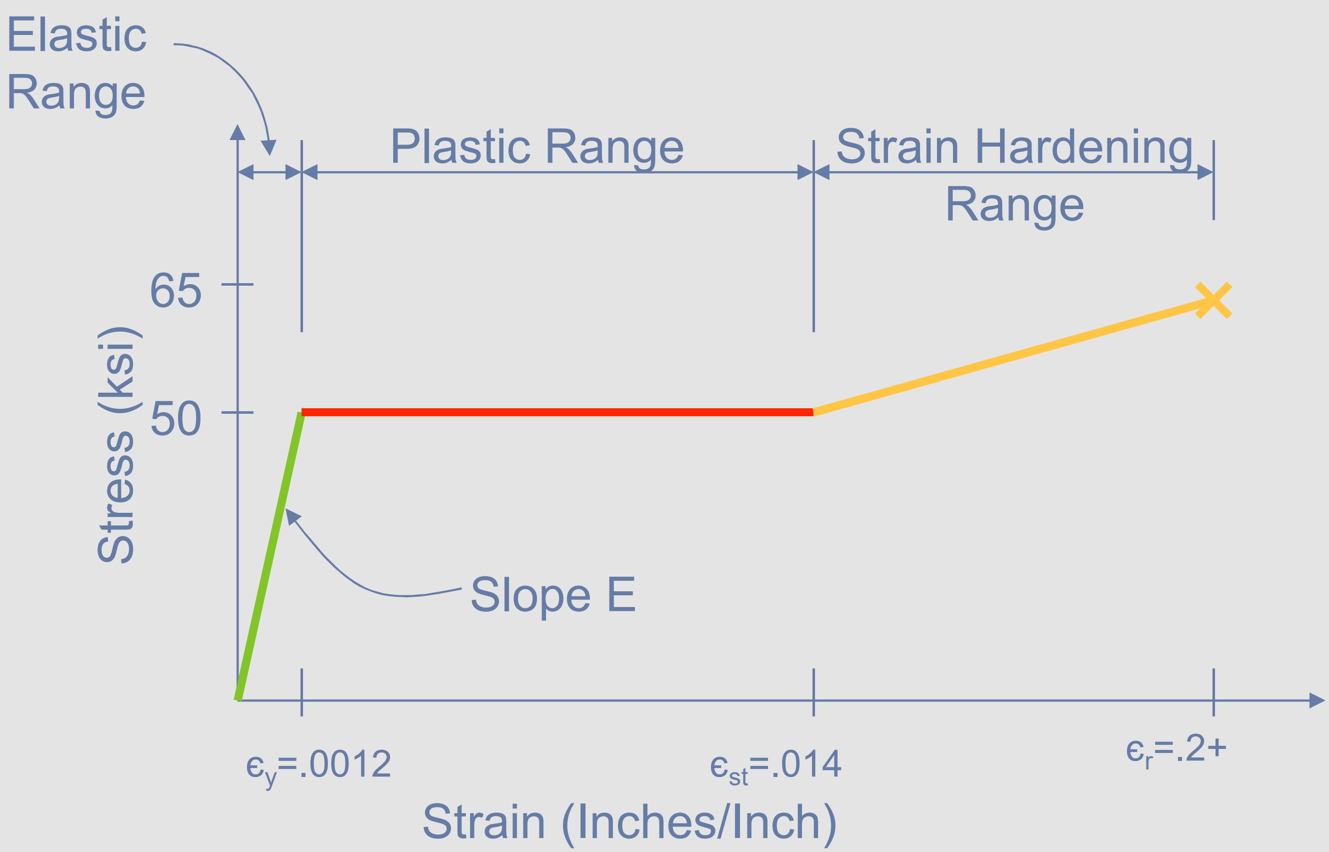
- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

# Flexural Stress and the Plastic Moment




As a beam receives increasing amounts of load, the stresses in the beam move from elastic to plastic. The plastic stresses overtake the elastic stresses for the entire depth of the shape. When this occurs, the beam has reached its plastic moment capacity.

# Idealized Stress-Strain Curve for Grade 50 Steel

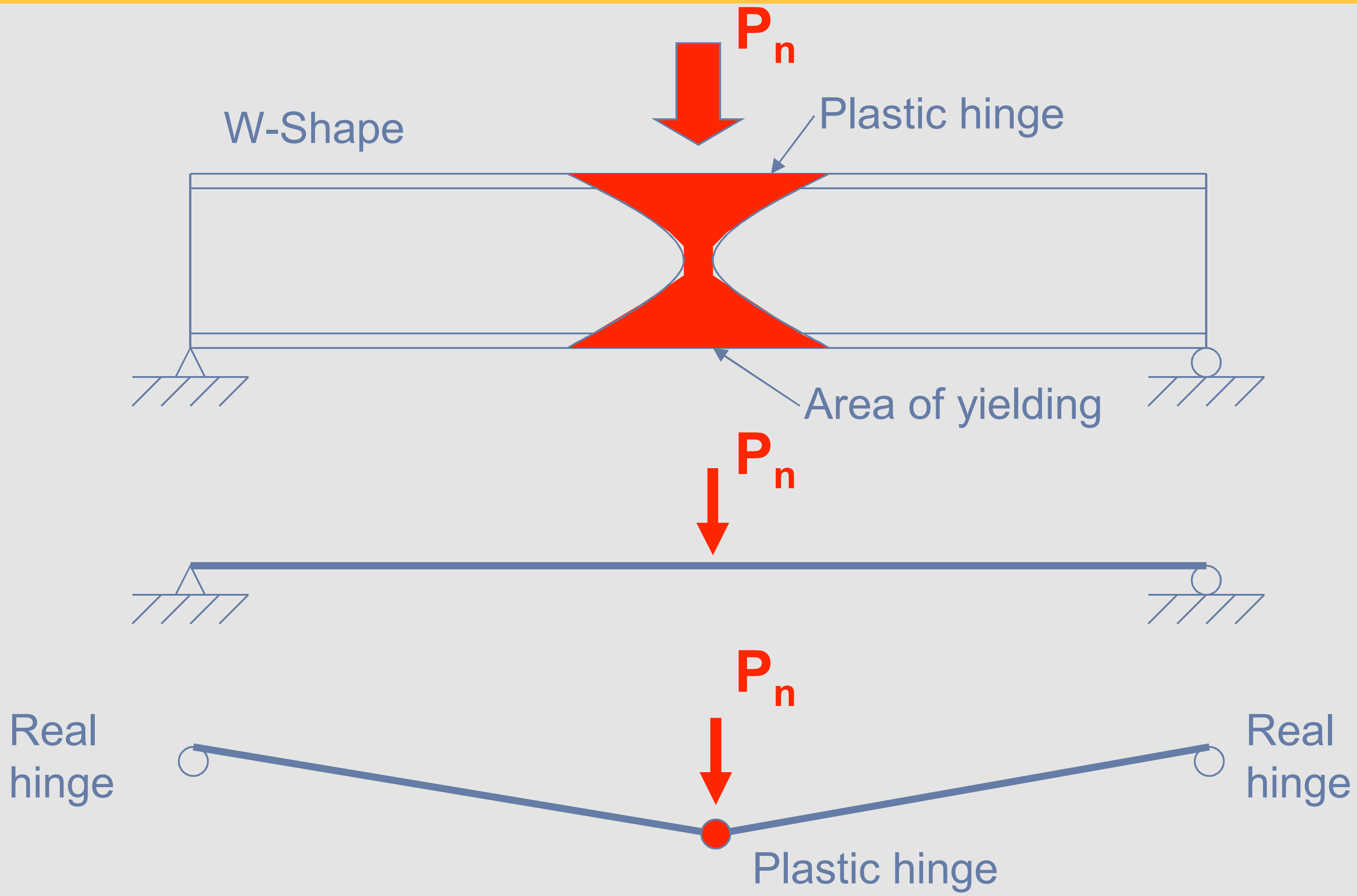


# Plastic Moment

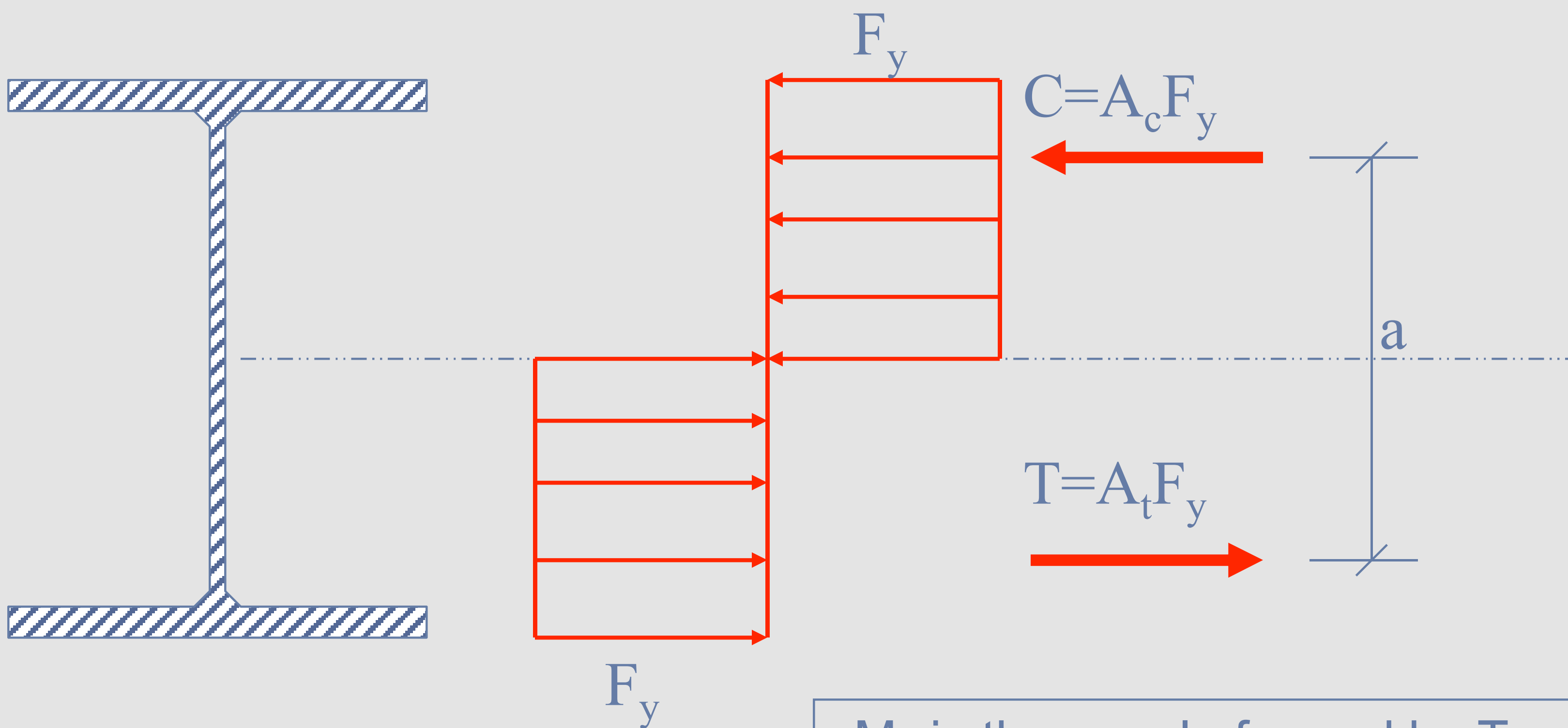
**Actual Hinging**

**Idealize Loading**

**Idealized Hinging**



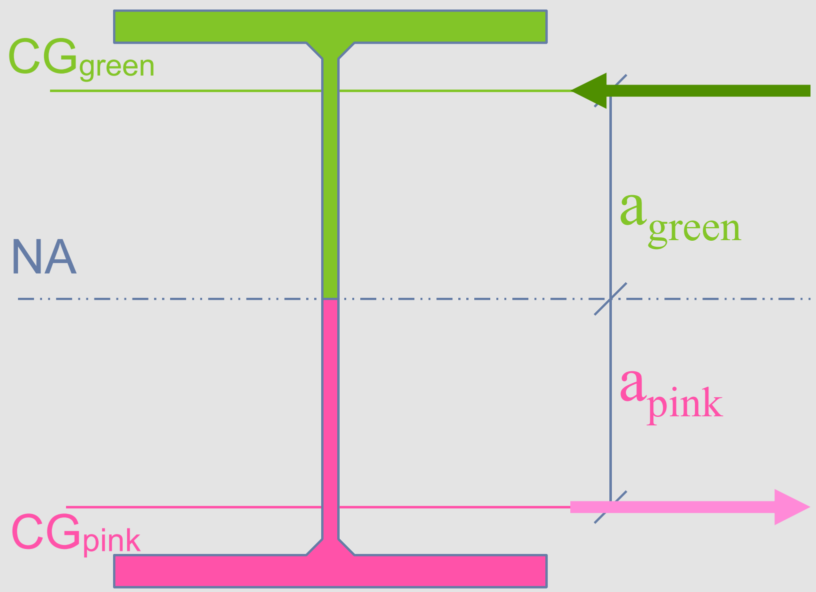
# Calculating the Plastic Moment ($M_p$)




- $M_p$ is the couple formed by T and C
- $M_p = F_y Z$ where:
  - Z = Plastic Section Modulus

# Plastic Section Modulus (Z)

1. Calculate the neutral axis of the entire section
2. Calculate the centroid of each piece above and below the neutral axis
3. $M_p = F_y*(A_{green}*a_{green} + A_{pink}*a_{pink})$
4. $Z = A_{green}*a_{green} + A_{pink}*a_{pink}$
5. $M_p = F_y Z$



# Nominal Plastic Flexural Capacity

**F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS**

This section applies to doubly symmetric I-shaped members and channels bent about their major axis, having compact webs and compact flanges as defined in Section B4.1 for flexure.

> **User Note:** All current ASTM A6 W, S, M, C and MC shapes except W21×48, W14×99, W14×90, W12×65, W10×12, W8×31, W8×10, W6×15, W6×9, W6×8.5 and M4×6 have compact flanges for $F_y$ = 50 ksi (345 MPa); all current ASTM A6 W, S, M, HP, C and MC shapes have compact webs at $F_y \leq$ 65 ksi (450 MPa).

The *nominal flexural strength*, $M_n$, shall be the lower value obtained according to the *limit states* of *yielding (plastic moment)* ~~and *lateral-torsional buckling*~~.

(LTB does not apply with continuous bracing)

# Nominal Plastic Flexural Capacity

**1. Yielding**

$$M_n = M_p = F_y Z_x \qquad \text{(F2-1)}$$

where

$F_y$ = *specified minimum yield stress* of the type of steel being used, ksi (MPa)

$Z_x$ = plastic section modulus about the $x$-axis, in.$^3$ (mm$^3$)

**Spec F2**

# Basic Design Equations for Beams

- $M_u \leq \Phi_b M_n$

  Where:

  - $M_u$ = Required Moment Strength
  - $M_n$ = Nominal Moment Strength
  - $\Phi_b = 0.9$

# Do We Need All These Equations?

- Refer to AISC SCM
  - Section Properties
    - Table 1-1:  lists S, I, Z
  - Beam Tables
    - Table 3-2:  select most economical size based on $Z_x$
    - Table 3-3:  select most economical size based on $I_x$

# Beam Tables Based on $Z_x$

- Strong Axis Plastic Section Modulus
- Very Commonly Used Table

$F_y$ = 50 ksi

**Table 3-2 (continued)**
**W-Shapes**
**Selection by $Z_x$**

$Z_x$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| **W30×116** | **378** | **943** | **1420** | **575** | **864** | **24.8** | **37.4** | **7.74** | **22.6** | **4930** | **339** | **509** |
| W21×147 | 373 | 931 | 1400 | 575 | 864 | 13.7 | 20.7 | 10.4 | 36.3 | 3630 | 318 | 477 |
| W24×131 | 370 | 923 | 1390 | 575 | 864 | 16.3 | 24.6 | 10.5 | 31.9 | 4020 | 296 | 445 |
| W18×158 | 356 | 888 | 1340 | 541 | 814 | 10.5 | 15.9 | 9.68 | 42.8 | 3060 | 319 | 479 |
| W14×193 | 355 | 886 | 1330 | 541 | 814 | 5.30 | 7.93 | 14.3 | 79.4 | 2400 | 276 | 414 |
| W12×210 | 348 | 868 | 1310 | 510 | 767 | 4.25 | 6.45 | 11.6 | 95.8 | 2140 | 347 | 520 |
| **W30×108** | **346** | **863** | **1300** | **522** | **785** | **23.5** | **35.5** | **7.59** | **22.1** | **4470** | **325** | **487** |
| W27×114 | 343 | 856 | 1290 | 522 | 785 | 21.7 | 32.8 | 7.70 | 23.1 | 4080 | 311 | 467 |
| W21×132 | 333 | 831 | 1250 | 515 | 774 | 13.2 | 19.9 | 10.3 | 34.2 | 3220 | 283 | 425 |
| W24×[illegible] | [illegible] | 816 | [illegible] | [illegible] | [illegible] | 15.4 | [illegible] | [illegible] | [illegible] | [illegible] | 267 | 401 |
| W18×143 | 322 | 803 | 1210 | 493 | 740 | 10.3 | 15.7 | 9.61 | 39.6 | 2750 | 285 | 427 |
| W14×176 | 320 | 798 | 1200 | 491 | 738 | 5.20 | 7.83 | 14.2 | 73.2 | 2140 | 252 | 378 |
| **W30×99** | **312** | **778** | **1170** | **470** | **706** | **22.2** | **33.4** | **7.42** | **21.3** | **3990** | **309** | **463** |
| W12×190 | 311 | 776 | 1170 | 459 | 690 | 4.18 | 6.33 | 11.5 | 87.3 | 1890 | 305 | 458 |
| W21×122 | 307 | 766 | 1150 | 477 | 717 | 12.9 | 19.3 | 10.3 | 32.7 | 2960 | 260 | 391 |
| W27×102 | 305 | 761 | 1140 | 466 | 701 | 20.1 | 29.8 | 7.59 | 22.3 | 3620 | 279 | 419 |
| W18×130 | 290 | 724 | 1090 | 447 | 672 | 10.2 | 15.4 | 9.54 | 36.6 | 2460 | 259 | 388 |
| W24×104 | 289 | 721 | 1080 | 451 | 677 | 14.3 | 21.3 | 10.3 | 29.2 | 3100 | 241 | 362 |
| W14×159 | 287 | 716 | 1080 | 444 | 667 | 5.17 | 7.85 | 14.1 | 66.7 | 1900 | 224 | 335 |
| **W30×90**$^v$ | **283** | **706** | **1060** | **428** | **643** | **20.6** | **30.8** | **7.38** | **20.9** | **3610** | **249** | **374** |
| W24×103 | 280 | 699 | 1050 | 428 | 643 | 18.2 | 27.4 | 7.03 | 21.9 | 3000 | 270 | 404 |
| W21×111 | 279 | 696 | 1050 | 435 | 654 | 12.4 | 18.9 | 10.2 | 31.2 | 2670 | 237 | 355 |
| W27×94 | 278 | 694 | 1040 | 424 | 638 | 19.1 | 28.5 | 7.49 | 21.6 | 3270 | 264 | 395 |
| W12×170 | 275 | 686 | 1030 | 410 | 617 | 4.11 | 6.15 | 11.4 | 78.5 | 1650 | 269 | 403 |
| W18×119 | 262 | 654 | 983 | 403 | 606 | 10.1 | 15.2 | 9.50 | 34.3 | 2190 | 249 | 373 |
| W14×145 | 260 | 649 | 975 | 405 | 609 | 5.13 | 7.69 | 14.1 | 61.7 | 1710 | 201 | 302 |
| W24×94 | 254 | 634 | 953 | 388 | 583 | 17.3 | 26.0 | 6.99 | 21.2 | 2700 | 250 | 375 |
| W21×101 | 253 | 631 | 949 | 396 | 596 | 11.8 | 17.7 | 10.2 | 30.1 | 2420 | 214 | 321 |
| **W27×84** | **244** | **609** | **915** | **372** | **559** | **17.6** | **26.4** | **7.31** | **20.8** | **2850** | **246** | **368** |
| W12×152 | 243 | 606 | 911 | 365 | 549 | 4.06 | 6.10 | 11.3 | 70.6 | 1430 | 238 | 358 |
| W14×132 | 234 | 584 | 878 | 365 | 549 | 5.15 | 7.74 | 13.3 | 55.8 | 1530 | 190 | 284 |
| W18×106 | 230 | 574 | 863 | 356 | 536 | 9.73 | 14.6 | 9.40 | 31.8 | 1910 | 221 | 331 |

| ASD | LRFD |
|---|---|
| $\Omega_b = 1.67$ | $\phi_b = 0.90$ |
| $\Omega_v = 1.50$ | $\phi_v = 1.00$ |

$^v$ Shape does not meet the $h/t_w$ limit for shear in AISC *Specification* Section G2.1(a) with $F_y$ = 50 ksi; therefore, $\phi_v$ = 0.90 and $\Omega_v$ = 1.67.

$F_y = 50$ ksi

# Table 3-2 (continued)
# W-Shapes
## Selection by $Z_x$

$Z_x$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF/\Omega_b$ | $\phi_b BF$ | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| **W30×116** | **378** | **943** | **1420** | **575** | **864** | **24.8** | **37.4** | **7.74** | **22.6** | **4930** | **339** | **509** |
| W21×147 | 373 | 931 | 1400 | 575 | 864 | 13.7 | 20.7 | 10.4 | 36.3 | 3630 | 318 | 477 |
| W24×131 | 370 | 923 | 1390 | 575 | 864 | 16.3 | 24.6 | 10.5 | 31.9 | 4020 | 296 | 445 |
| W18×158 | 356 | 888 | 1340 | 541 | 814 | 10.5 | 15.9 | 9.68 | 42.8 | 3060 | 319 | 479 |
| W14×193 | 355 | 886 | 1330 | 541 | 814 | 5.30 | 7.93 | 14.3 | 79.4 | 2400 | 276 | 414 |
| W12×210 | 348 | 868 | 1310 | 510 | 767 | 4.25 | 6.45 | 11.6 | 95.8 | 2140 | 347 | 520 |
| **W30×108** | **346** | **863** | **1300** | **522** | **785** | **23.5** | **35.5** | **7.59** | **22.1** | **4470** | **325** | **487** |
| W27×114 | 343 | 856 | 1290 | 522 | 785 | 21.7 | 32.8 | 7.70 | 23.1 | 4080 | 311 | 467 |
| W21×132 | 333 | 831 | 1250 | 515 | 774 | 13.2 | 19.9 | 10.3 | 34.2 | 3220 | 283 | 425 |
| W24×117 | 327 | 816 | 1230 | 508 | 764 | 15.4 | 23.3 | 10.4 | 30.4 | 3540 | 267 | 401 |
| W18×143 | 322 | 803 | 1210 | 493 | 740 | 10.3 | 15.7 | 9.61 | 39.6 | 2750 | 285 | 427 |
| W14×176 | 320 | 798 | 1200 | 491 | 738 | 5.20 | 7.83 | 14.2 | 73.2 | 2140 | 252 | 378 |

| ASD | LRFD |
|---|---|
| $\Omega_b = 1.67$<br>$\Omega_v = 1.50$ | $\phi_b = 0.90$<br>$\phi_v = 1.00$ |

[v] Shape does not meet the $h/t_w$ limit for shear in AISC *Specification* Section G2.1(a) with $F_y = 50$ ksi; therefore, $\phi_v = 0.90$ and $\Omega_v = 1.67$.

# Beam Tables Based on $I_x$

- Strong Axis Moment of Inertia
- Very Commonly Used Table

Table 3-3 (continued)

**W-Shapes**

Selection by $I_x$

$I_x$

| Shape | $I_x$ in.$^4$ | Shape | $I_x$ in.$^4$ | Shape | $I_x$ in.$^4$ | Shape | $I_x$ in.$^4$ |
|---|---|---|---|---|---|---|---|
| **W30×90** | **3610** | **W24×68** | **1830** | **W21×44** | **843** | **W16×26** | **301** |
| W12×305[h] | 3550 | W21×83 | 1830 | W12×96 | 833 | W14×30 | 291 |
| W24×117 | 3540 | W18×97 | 1750 | W18×50 | 800 | W12×35 | 285 |
| W18×175 | 3450 | W14×145 | 1710 | W14×74 | 795 | W10×49 | 272 |
| W14×257 | 3400 | W12×170 | 1650 | W16×57 | 758 | W8×67 | 272 |
| W27×94 | 3270 | W21×73 | 1600 | W12×87 | 740 | W10×45 | 248 |
| W21×132 | 3220 | | | W14×68 | 722 | | |
| W12×279[h] | 3110 | **W24×62** | **1550** | W10×112 | 716 | **W14×26** | **245** |
| W24×104 | 3100 | W18×86 | 1530 | W18×46 | 712 | W12×30 | 238 |
| W18×158 | 3060 | W14×132 | 1530 | W12×79 | 662 | W8×58 | 228 |
| W14×233 | 3010 | W16×100 | 1490 | W16×50 | 659 | W10×39 | 209 |
| [illegible]103 | 3000 | W21×[illegible] | [illegible] | W14×61 | 64[illegible] | | |
| W21×122 | 2960 | W12×152 | 1430 | W10×100 | 623 | **W12×26** | **204** |
| | | W14×120 | 1380 | | | | |
| **W27×84** | **2850** | | | **W18×40** | **612** | **W14×22** | **199** |
| W18×143 | 2750 | **W24×55** | **1350** | W12×72 | 597 | W8×48 | 184 |
| W12×252[h] | 2720 | W21×62 | 1330 | W16×45 | 586 | W10×33 | 171 |
| W24×94 | 2700 | W18×76 | 1330 | W14×53 | 541 | W10×30 | 170 |
| W21×111 | 2670 | W16×89 | 1300 | W10×88 | 534 | | |
| W14×211 | 2660 | W14×109 | 1240 | W12×65 | 533 | **W12×22** | **156** |
| W18×130 | 2460 | W12×136 | 1240 | | | W8×40 | 146 |
| W21×101 | 2420 | W21×57 | 1170 | **W16×40** | **518** | W10×26 | 144 |
| W12×230[h] | 2420 | W18×71 | 1170 | | | | |
| W14×193 | 2400 | | | **W18×35** | **510** | **W12×19** | **130** |
| | | **W21×55** | **1140** | W14×48 | 484 | W8×35 | 127 |
| **W24×84** | **2370** | W16×77 | 1110 | W12×58 | 475 | W10×22 | 118 |
| W18×119 | 2190 | W14×99 | 1110 | W10×77 | 455 | W8×31 | 110 |
| W14×176 | 2140 | W18×65 | 1070 | W16×36 | 448 | | |
| W12×210 | 2140 | W12×120 | 1070 | W14×43 | 428 | **W12×16** | **103** |
| | | W14×90 | 999 | W12×53 | 425 | W8×28 | 98.0 |
| **W24×76** | **2100** | | | W10×68 | 394 | W10×19 | 96.3 |
| W21×93 | 2070 | **W21×50** | **984** | W12×50 | 391 | | |
| W18×106 | 1910 | W18×60 | 984 | W14×38 | 385 | **W12×14** | **88.6** |
| W14×159 | 1900 | | | | | W8×24 | 82.7 |
| W12×190 | 1890 | **W21×48** | **959** | **W16×31** | **375** | W10×17 | 81.9 |
| | | W16×67 | 954 | W12×45 | 348 | W8×21 | 75.3 |
| | | W12×106 | 933 | W10×60 | 341 | W10×15 | 68.9 |
| | | W18×55 | 890 | W14×34 | 340 | W8×18 | 61.9 |
| | | W14×82 | 881 | W12×40 | 307 | | |
| | | | | W10×54 | 303 | **W10×12** | **53.8** |
| | | | | | | W8×15 | 48.0 |
| | | | | | | W8×13 | 39.6 |
| | | | | | | **W8×10** | **30.8** |

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

# Table 3-3 (continued)
# W-Shapes
## Selection by $I_x$

$I_x$

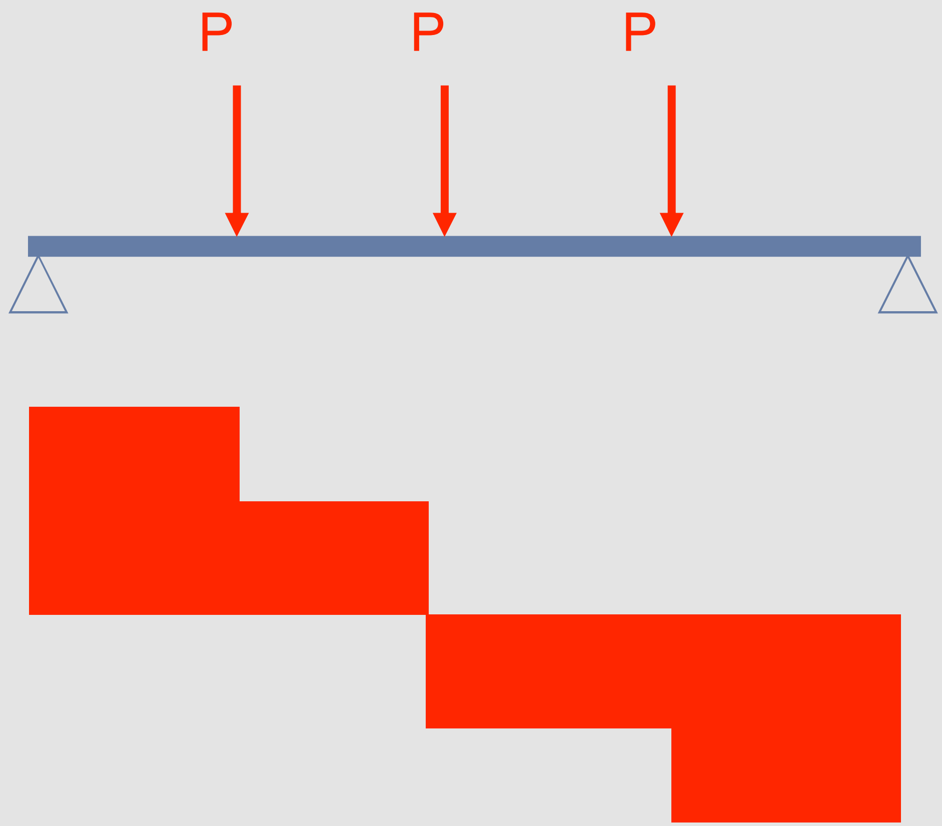
| Shape | $I_x$ in.$^4$ | Shape | $I_x$ in.$^4$ | Shape | $I_x$ in.$^4$ | Shape | $I_x$ in.$^4$ |
|---|---|---|---|---|---|---|---|
| **W30×90** | **3610** | **W24×68** | **1830** | **W21×44** | **843** | **W16×26** | **301** |
| W12×305$^h$ | 3550 | W21×83 | 1830 | W12×96 | 833 | W14×30 | 291 |
| W24×117 | 3540 | W18×97 | 1750 | W18×50 | 800 | W12×35 | 285 |
| W18×175 | 3450 | W14×145 | 1710 | W14×74 | 795 | W10×49 | 272 |
| W14×257 | 3400 | W12×170 | 1650 | W16×57 | 758 | W8×67 | 272 |
| W27×94 | 3270 | W21×73 | 1600 | W12×87 | 740 | W10×45 | 248 |
| W21×132 | 3220 | | | W14×68 | 722 | | |
| W12×279$^h$ | 3110 | **W24×62** | **1550** | W10×112 | 716 | **W14×26** | **245** |
| W24×104 | 3100 | W18×86 | 1530 | W18×46 | 712 | W12×30 | 238 |
| W18×158 | 3060 | W14×132 | 1530 | W12×79 | 662 | W8×58 | 228 |
| W14×233 | 3010 | W16×100 | 1490 | W16×50 | 659 | W10×39 | 209 |
| W24×103 | 3000 | W21×68 | 1480 | W14×61 | 640 | | |
| W21×122 | 2960 | W12×152 | 1430 | W10×100 | 623 | **W12×26** | **204** |
| | | W14×120 | 1380 | | | | |

# Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

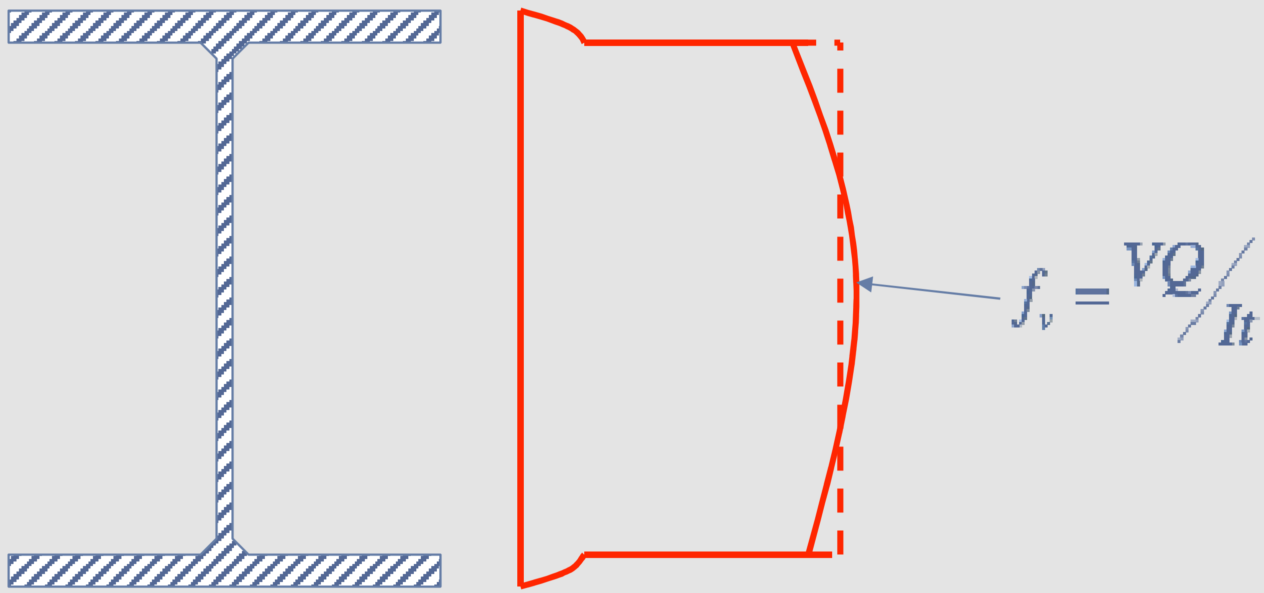
- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

# Calculation of Shear Stress




Shear Diagram

# Examine Stress in a X-Sect




Clearly, $f_v$ in the web governs the design over that in the flange. Also note that the average $f_v$ is almost equal to the maximum $f_v$.

Therefore, we can re-define $f_v$ in an easier way…

# Basic Shear Strength Relationships

- Spec Chapter G: Design of Members for Shear
  - $V_u \leq \Phi_v V_n$ where:
    - $V_u$ = Required shear strength
    - $\Phi_v$ = Reduction factor for shear
    - $V_n$ = Nominal shear strength

- General case:
  - $V_n = 0.6\ F_y A_w C_v$ (Eq G2-1)
  - $\Phi_v = 0.90$
  - $C_v$ accounts for the web slenderness

# Shear Limit States

- There are different shear limit states depending on the mode of shear failure
  - Shear yielding
  - Web buckling
- These limit states are dependent on the slenderness ratio of the web, $h/t_w$.
  - The value of $C_v$ in Eq G2-1 varies depending on the value of $h/t_w$.
  - Generally, more slender web ➔ smaller $C_v$

# Typical W-Shape Shear Strength

- However, for webs of rolled I-shape members with

$$h/t_w \leq 2.24\sqrt{E/F_y}$$

  - $\Phi_v = 1.00$
  - $C_v = 1.0$

- This applies to almost every rolled steel shape

**User Note:** All current ASTM A6 W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 meet the criteria stated in Section G2.1(a) for $F_y$ = 50 ksi (345 MPa).

- See Spec G2.1(a)

# AISC: Shear Strength

(b) For webs of all other doubly symmetric shapes and singly symmetric shapes and channels, except round *HSS*, the web shear coefficient, $C_v$, is determined as follows:

(i) When $h/t_w \le 1.10\sqrt{k_v E/F_y}$

$$C_v = 1.0 \qquad \text{(G2-3)}$$

(ii) When $1.10\sqrt{k_v E/F_y} < h/t_w \le 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.10\sqrt{k_v E/F_y}}{h/t_w} \qquad \text{(G2-4)}$$

(iii) When $h/t_w > 1.37\sqrt{k_v E/F_y}$

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \qquad \text{(G2-5)}$$

**Spec G2.1**

# AISC: Shear Strength

The web plate *shear buckling* coefficient, $k_v$, is determined as follows:

(i) For webs without *transverse stiffeners* and with $h/t_w < 260$:

$$k_v = 5$$

except for the stem of tee shapes where $k_v = 1.2$.

(ii) For webs with transverse stiffeners:

$$k_v = 5 + \frac{5}{(a/h)^2} \tag{G2-6}$$

$$= 5 \text{ when } a/h > 3.0 \text{ or } a/h > \left[\frac{260}{(h/t_w)}\right]^2$$

where

$a$ = clear distance between transverse stiffeners, in. (mm)

**User Note:** For all ASTM A6 W, S, M and HP shapes except M12.5×12.4, M12.5×11.6, M12×11.8, M12×10.8, M12×10, M10×8 and M10×7.5, when $F_y$ = 50 ksi (345 MPa), $C_v = 1.0$.

# Limit States for Flexure

- Plastic Flexural Capacity
- Shear Capacity
- Deflection

- *Only limited to these three limit states when:*
  - *compression flange is laterally braced*
  - *beam is compact*

# Serviceability

- Serviceability is related to performance.
- A design must not only have sufficient strength, but also must not cause significant discomfort for users.
- Deflections and vibrations are the most common serviceability considerations.
- As such, we impose limits on <u>service load</u> deflections which will provide good serviceability.

# Deflection Criteria

- Typical Criteria:
  - Live Load Deflection Held to Less Than L/360
  - Dead + Live Load Deflection Held to Less than L/240.
  - L is Measured in Inches.
  - Loads Are Service Loads!

**LOADS ARE SERVICE LOADS!**

# Summary of Beam Design with Lateral Support

1. Establish Loading (Assuming self-weight)
2. Find $M_u$
3. Use $Z_{x\ rqd.} = M_u/\Phi_b F_y$; Use Table 3-2 to select a section with required $Z_x$.
4. Check assumption of beam weight; Iterate as required.
5. Check Shear; Iterate as required.
6. Check Deflection; Iterate as required.

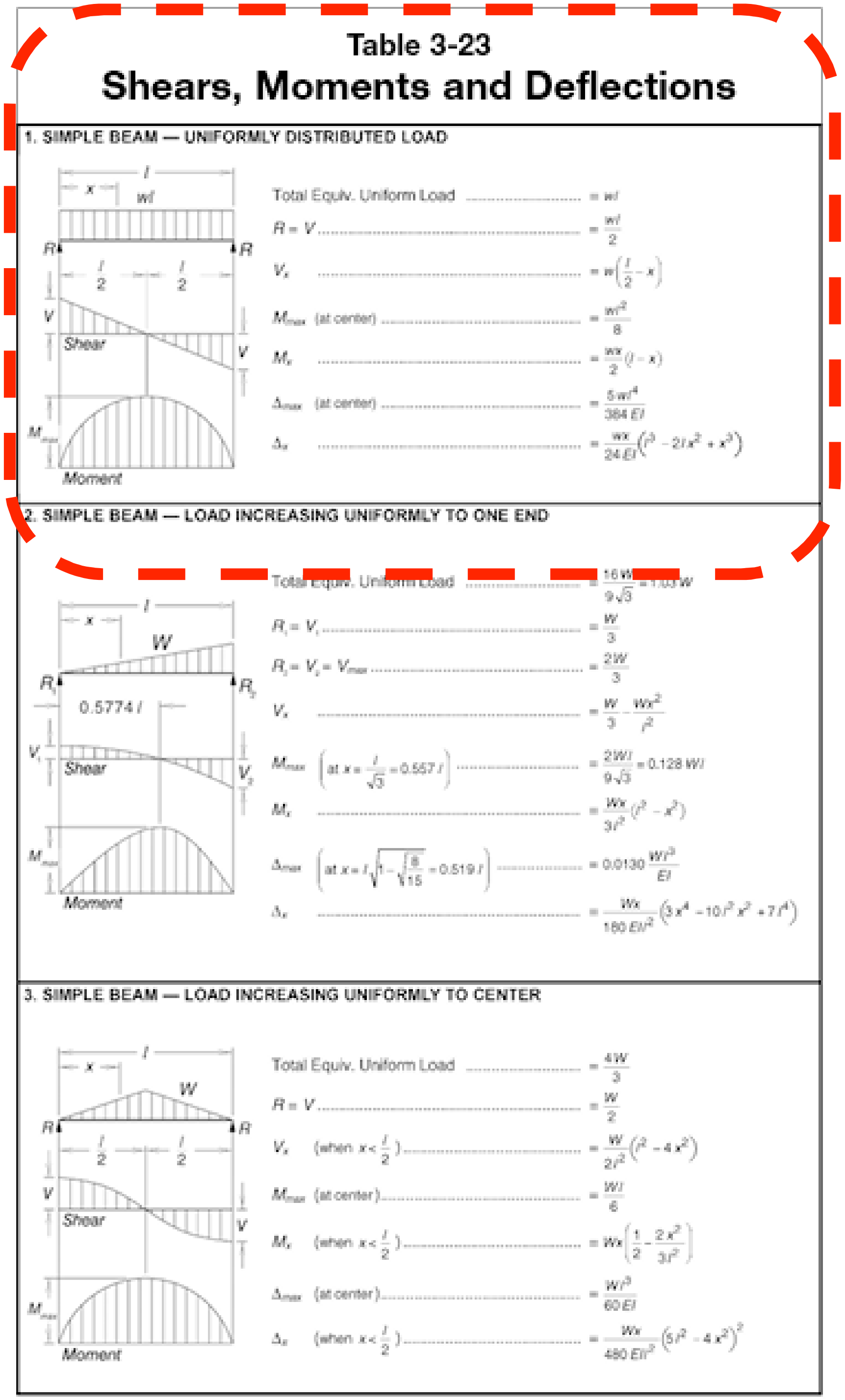


**Table 3-23**
**Shears, Moments and Deflections**

**1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD**

Total Equiv. Uniform Load $= wl$

$R = V = \frac{wl}{2}$

$V_x = w\left(\frac{l}{2} - x\right)$

$M_{max}$ (at center) $= \frac{wl^2}{8}$

$M_x = \frac{wx}{2}(l - x)$

$\Delta_{max}$ (at center) $= \frac{5wl^4}{384EI}$

$\Delta_x = \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

**2. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO ONE END**

Total Equiv. Uniform Load $= \frac{16W}{9\sqrt{3}} = 1.03W$

$R_1 = V_1 = \frac{W}{3}$

$R_2 = V_2 = V_{max} = \frac{2W}{3}$

$V_x = \frac{W}{3} - \frac{Wx^2}{l^2}$

$M_{max}$ $\left(\text{at } x = \frac{l}{\sqrt{3}} = 0.577\,l\right)$ $= \frac{2Wl}{9\sqrt{3}} = 0.128\,Wl$

$M_x = \frac{Wx}{3l^2}\left(l^2 - x^2\right)$

$\Delta_{max}$ $\left(\text{at } x = l\sqrt{1 - \sqrt{\frac{8}{15}}} = 0.519\,l\right)$ $= 0.0130\frac{Wl^3}{EI}$

$\Delta_x = \frac{Wx}{180EIl^2}\left(3x^4 - 10l^2x^2 + 7l^4\right)$

**3. SIMPLE BEAM — LOAD INCREASING UNIFORMLY TO CENTER**

Total Equiv. Uniform Load $= \frac{4W}{3}$

$R = V = \frac{W}{2}$

$V_x$ $\left(\text{when } x < \frac{l}{2}\right)$ $= \frac{W}{2l^2}\left(l^2 - 4x^2\right)$

$M_{max}$ (at center) $= \frac{Wl}{6}$

$M_x$ $\left(\text{when } x < \frac{l}{2}\right)$ $= Wx\left(\frac{1}{2} - \frac{2x^2}{3l^2}\right)$

$\Delta_{max}$ (at center) $= \frac{Wl^3}{60EI}$

$\Delta_x$ $\left(\text{when } x < \frac{l}{2}\right)$ $= \frac{Wx}{480EIl^2}\left(5l^2 - 4x^2\right)^2$

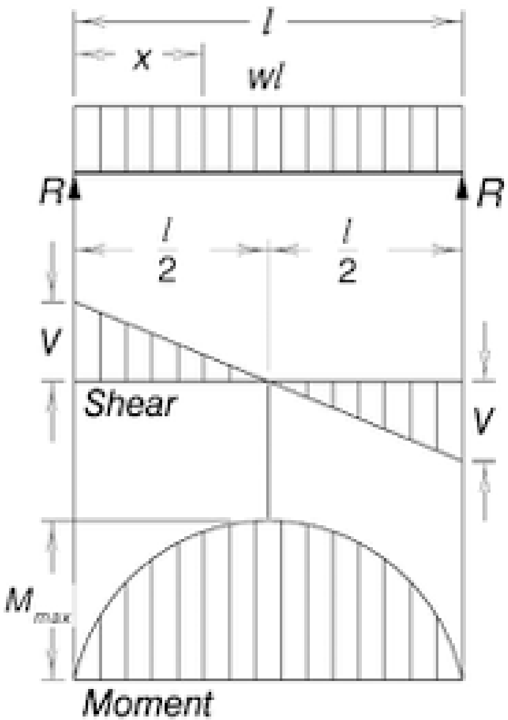
# Tools for Calculating Internal Forces & Deflections

- Most loading conditions encountered in design are solved in the manual.
- HUGE TIME SAVER

# Table 3-23

# Shears, Moments and Deflections

## 1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD




Total Equiv. Uniform Load ............................ $= wl$

$R = V$ ............................ $= \dfrac{wl}{2}$

$V_x$ ............................ $= w\left(\dfrac{l}{2} - x\right)$

$M_{max}$ (at center) ............................ $= \dfrac{wl^2}{8}$

$M_x$ ............................ $= \dfrac{wx}{2}(l - x)$

$\Delta_{max}$ (at center) ............................ $= \dfrac{5\,wl^4}{384\,EI}$

$\Delta_x$ ............................ $= \dfrac{wx}{24\,EI}\left(l^3 - 2lx^2 + x^3\right)$

# Table 3-22c

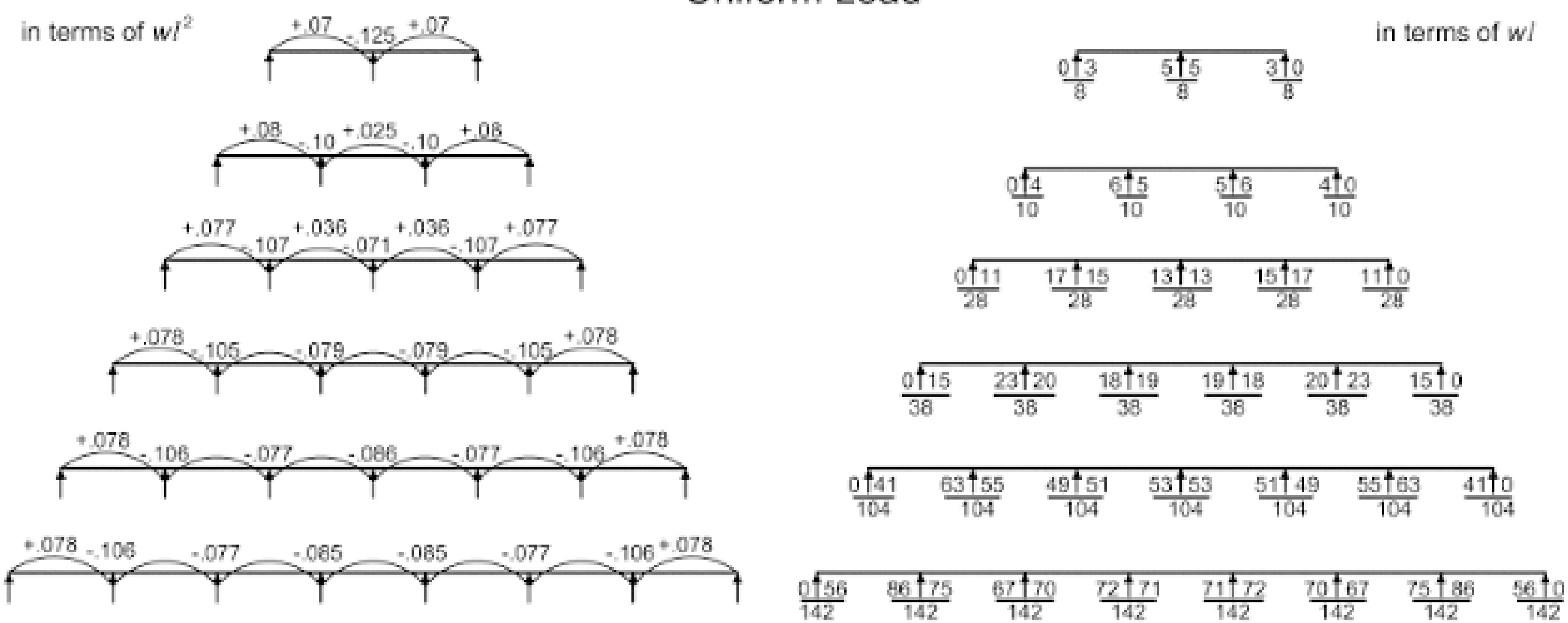
# Continuous Beams

## Moments and Shear Coefficients—Equal Spans, Equally Loaded

**Moment** in terms of $wl^2$

**Uniform Load**

**Shear** in terms of $wl$



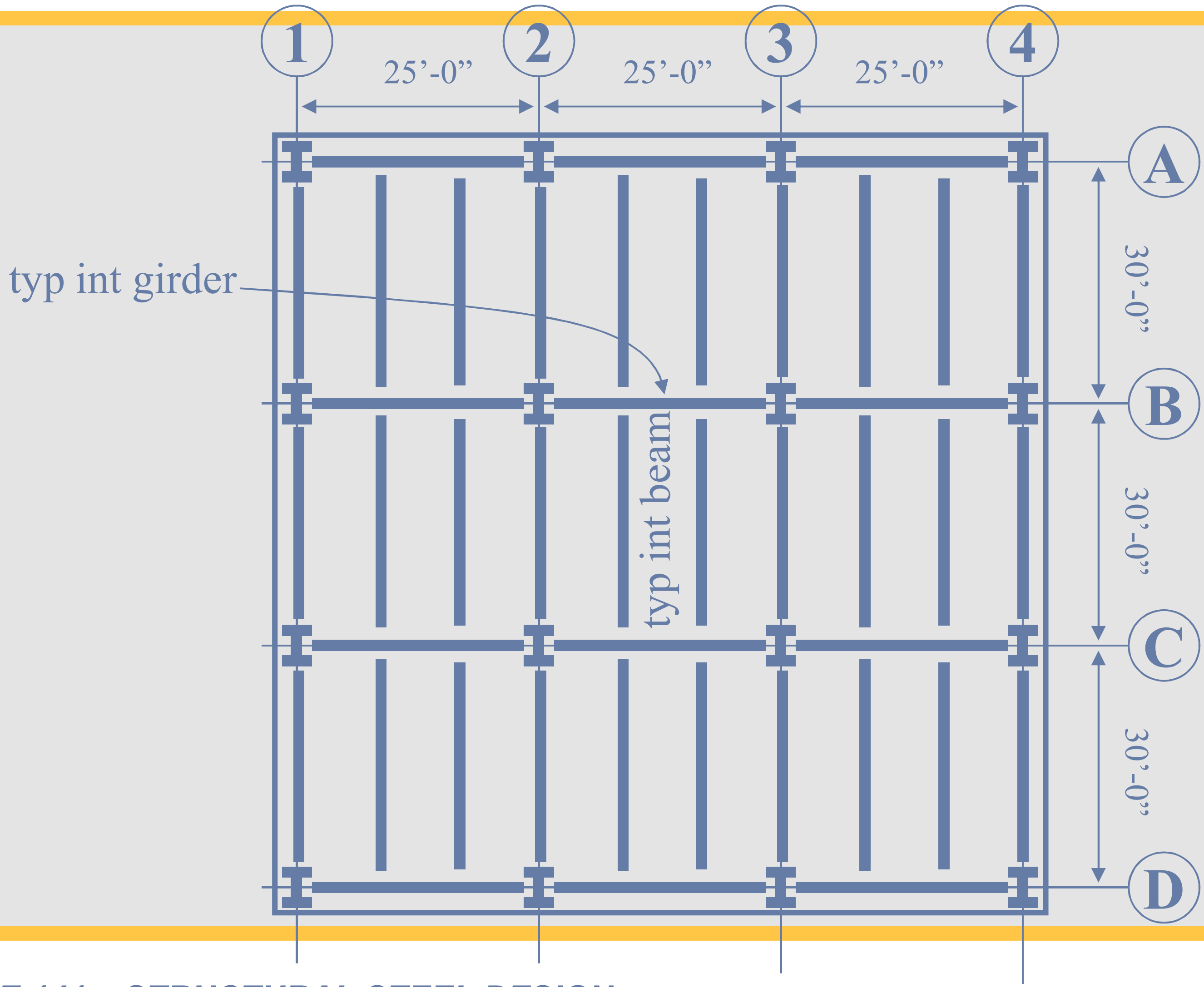
# Questions?

# Example Problem

- Design the most efficient typical interior WF beam.
- Design the most efficient typical interior WF girder.

- Assumptions:
  - Floor Plans are on the following page
  - Floor Loading: Dead = 100psf, Live = 40 psf (reducible)
  - Self weight of the members is included in the dead load
  - A992 Steel
  - Beam compression flanges are fully braced by headed studs to concrete fill on metal deck
  - Deflection Limits: Dead + Live = L/240, Live = L/360
  - Live Load Reducible per ASCE 7-10

# Example Problem




**Floor Framing Plan**

# Example Problem

- **Typical Interior Beam**

  - Calculate Loads

    $DL = 100$ psf; $LL_0 = 40$ psf

    Trib Area = (25'/3) (30') = 250 ft$^2$

    $LL = LL_0\,[0.25 + 15 / \mathrm{sqrt}(K_{LL}\,A_T)]$

    $= (40\text{psf})\,[0.25 + 15 / \mathrm{sqrt}\,(2 \times 250\text{ft}^2)]$

    $= 37$ psf

# Example Problem

- Calculate Applied Loads
  - $w_s$ = DL + LL = (100psf + 37psf) (25'/3)
    = 1.14klf
  - $w_L$ = LL = (37psf) (25'/3)
    = 0.31klf
  - $w_u$ = 1.2DL + 1.6LL
    = [(1.2) (100psf) + (1.6) (37psf)] (25'/3)
    = 1.49 klf
- Calculate Internal Forces

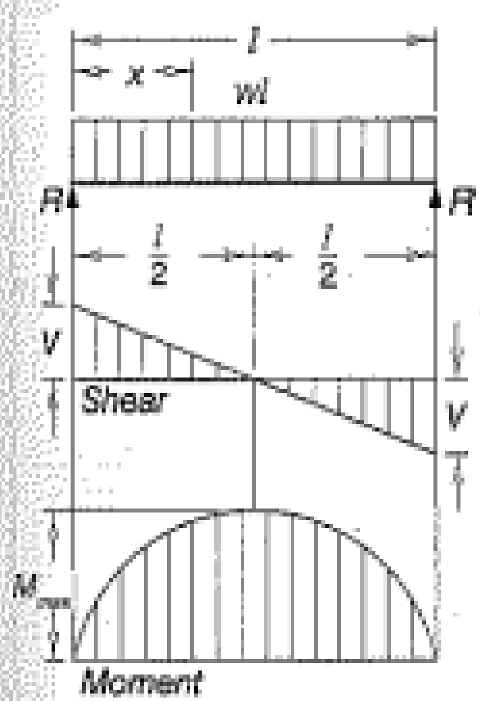
$$M_u = w_u L^2 / 8 = (1.49\text{klf}) (30')^2 / 8 = 168\text{k-ft}$$

$$V_u = w_u L / 2 = (1.49\text{klf}) (30') / 2 = 22.4\text{k}$$

# Example Problem

- Table 3-23 (pg 3-211)

**1. SIMPLE BEAM — UNIFORMLY DISTRIBUTED LOAD**




Total Equiv. Uniform Load ........ $= wl$

$R = V$ ........ $= \frac{wl}{2}$

$V_x$ ........ $= w\left(\frac{l}{2} - x\right)$

$M_{max}$ (at center) ........ $= \frac{wl^2}{8}$

$M_x$ ........ $= \frac{wx}{2}(l - x)$

$\Delta_{max}$ (at center) ........ $= \frac{5wl^4}{384EI}$

$\Delta_x$ ........ $= \frac{wx}{24EI}\left(l^3 - 2lx^2 + x^3\right)$

# Example Problem

- Check Flexural Capacity

  Top Flange Fully Braced ($L_b$ = 0')

  $Z_{reqd} = M_u / \phi_b F_y$

  = (168k-ft) (12"/ft) / (0.9) (50 ksi)

  = 44.8 $in^3$

- ***Try W14x30*** (Z = 47.3 $in^3$)

- Check Shear Capacity

  $\phi_v V_n = \phi_v (0.6) A_w F_y C_v$

  = (1.0) (0.6) (13.84") (0.27") (50ksi) (1.0)

  = 121k *therefore OK for shear*

$Z_x$

## Table 3-2 (continued)
# W Shapes
### Selection by $Z_x$

$F_y$ = 50 ksi

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | BF | | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| W18×35 | 66.5 | 166 | 249 | 101 | 151 | 8.07 | 12.1 | 4.31 | 12.4 | 510 | 106 | 159 |
| W14×30 | 47.3 | 118 | 177 | 73.4 | 110 | 4.65 | 6.99 | 5.26 | 14.9 | 291 | 74.7 | 112 |
| W10×39 | 46.8 | 117 | 176 | 73.5 | 111 | 2.51 | 3.77 | 6.99 | 24.2 | 209 | 62.5 | 93.7 |
| W16×26[v] | 44.2 | 110 | 166 | 67.1 | 101 | 5.96 | 8.96 | 3.96 | 11.2 | 301 | 70.5 | 106 |
| W14×26 | 40.2 | 100 | 151 | 61.7 | 92.7 | 5.32 | 7.99 | 3.81 | 11.1 | 245 | 70.9 | 106 |
| W8×40 | 39.8 | 99.3 | 149 | 62.0 | 93.2 | 1.64 | 2.47 | 7.21 | 29.9 | 146 | 59.4 | 89.1 |
| W10×33 | 38.8 | 96.8 | 146 | 61.1 | 91.9 | 2.39 | 3.59 | 6.85 | 21.8 | 171 | 56.4 | 84.7 |
| W12×26 | 37.2 | 92.8 | 140 | 58.3 | 87.7 | 3.61 | 5.42 | 5.33 | 14.9 | 204 | 56.2 | 84.3 |
| W10×30 | 36.6 | 91.3 | 137 | 56.6 | 85.0 | 3.08 | 4.62 | 4.84 | 16.1 | 170 | 62.8 | 94.2 |
| W8×35 | 34.7 | 86.6 | 130 | 54.5 | 81.9 | 1.62 | 2.43 | 7.17 | 27.0 | 127 | 50.3 | 75.5 |
| W14×22 | 33.2 | 82.8 | 125 | 50.6 | 76.1 | 4.75 | 7.14 | 3.67 | 10.4 | 199 | 63.2 | 94.8 |
| W10×26 | 31.3 | 78.1 | 117 | 48.7 | 73.2 | 2.90 | 4.36 | 4.80 | 14.9 | 144 | 53.7 | 80.6 |
| W8×31[f] | 30.4 | 75.8 | 114 | 48.0 | 72.2 | 1.58 | 2.37 | 7.18 | 24.8 | 110 | 45.6 | 68.4 |
| W12×22 | 29.3 | 73.1 | 110 | 44.4 | 66.7 | 4.65 | 6.99 | 3.00 | 9.17 | 156 | 64.0 | 96.0 |
| W8×28 | 27.2 | 67.9 | 102 | 42.4 | 63.8 | 1.66 | 2.50 | 5.72 | 21.0 | 98.0 | 45.9 | 68.9 |
| W10×22 | 26.0 | 64.9 | 97.5 | 40.5 | 60.9 | 2.68 | 4.02 | 4.70 | 13.8 | 118 | 48.8 | 73.2 |
| W12×19 | 24.7 | 61.6 | 92.6 | 37.2 | 55.9 | 4.27 | 6.43 | 2.90 | 8.62 | 130 | 57.2 | 85.7 |
| W8×24 | 23.1 | 57.6 | 86.6 | 36.5 | 54.9 | 1.59 | 2.39 | 5.69 | 19.0 | 82.7 | 38.9 | 58.3 |
| W10×19 | 21.6 | 53.9 | 81.0 | 32.8 | 49.3 | 3.17 | 4.77 | 3.09 | 9.72 | 96.3 | 51.2 | 76.8 |
| W8×21 | 20.4 | 50.9 | 76.5 | 31.8 | 47.8 | 1.86 | 2.79 | 4.45 | 14.8 | 75.3 | 41.4 | 62.1 |

| ASD | LRFD |
|---|---|
| $\Omega_b = 1.67$ | $\phi_b = 0.90$ |
| $\Omega_v = 1.50$ | $\phi_v = 1.00$ |

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

[v] Shape does not meet the $h/t_w$ limit for shear in Specification Section G2.1a with $F_y = 50$ ksi, $\Omega_v = 1.67$, $\phi_v = 0.90$.

# Example Problem

- Check Serviceability
  - Live Load Deflection

$$\Delta_{max} \quad \text{(at center)} \;\ldots\ldots\; = \frac{5wl^4}{384EI}$$

$\Delta_L \quad = 5\ w\ L^4\ /\ 384\ E\ I$

$= (5)(0.31\text{klf})(30')^4(12''/\text{ft})^3/(384)(29\text{E}3\text{ksi})\ (291\text{in}^4)$

$= 0.67'' < L/360 = (30')(12''/\text{ft})/360 = 1''$ *therefore OK*

  - Dead Load + Live Load Deflection

$\Delta_{D+L} \quad = 5\ w\ L^4\ /\ 384\ E\ I$

$= (5)(1.14\text{klf})(30')^4(12''/\text{ft})^3/(384)(29\text{E}3\text{ksi})\ (291\text{in}^4)$

$= 2.46'' < L/240 = (30')(12''/\text{ft})/240 = 1.5''$ *therefore NG*

# Example Problem

- Check Serviceability
  - Determine $I_{reqd}$

$I_{reqd}$ = [(2.46")/(1.5")] (291 $in^4$) = 477$in^4$

Options:

W18x35 (I = 510; Z = 66.5) *select lightest*

W16x40 (I = 518; Z = 72.9)

**Use W18x35 typical interior beam**

# Example Problem

- **Typical Interior Girder**

  - **Calculate Loads**

    DL = 100 psf; $LL_0$ = 40 psf

    Trib Area = (2/3) (25’) (30’) = 500 $ft^2$

    $LL = LL_0\,[0.25 + 15 / \text{sqrt}(K_{LL}\ A_T)]$

    $= (40psf)\,[0.25 + 15 / \text{sqrt}\ (2 \times 500ft^2)]$

    = 29 psf

# Example Problem

- Calculate Applied Loads
  - $P_D = (100\text{psf})\ (25'/3)\ (30') = 25.0\text{k}$
  - $P_L = (29\text{psf})\ (25'/3)\ (30') = 7.3\text{k}$
  - $P_s = P_D + P_L = 25.0\text{k} + 7.3\text{k} = 32.3\text{k}$
  - $P_u = 1.2P_D + 1.6P_L$
    $= (1.2)(25.0\text{k}) + (1.6)(7.3\text{k}) = 41.7\text{k}$
- Calculate Internal Forces

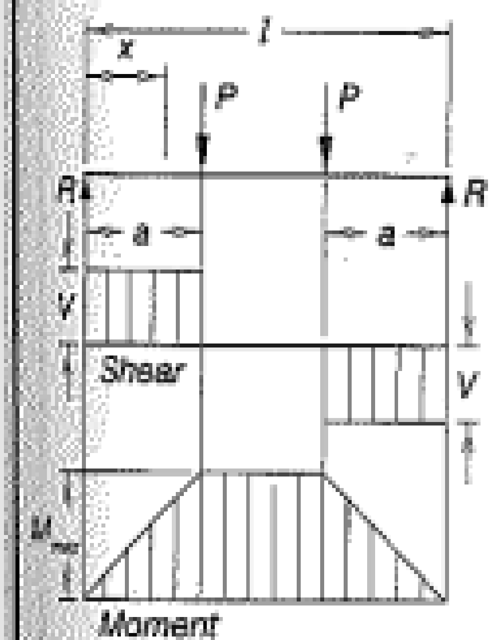
$$M_u = P_u\ L / 3 = (41.7\text{k})\ (25'/3) / 3 = 347\text{kft}$$

$$V_u = P_u = 41.7\text{k}$$

# Example Problem

- Table 3-23 (pg 3-213)

**9. SIMPLE BEAM — TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED**




| | | |
|---|---|---|
| Total Equiv. Uniform Load | ........ | $= \frac{8Pa}{l}$ |
| $R=V$ | ........ | $= P$ |
| $M_{max}$ (between loads) | ........ | $= Pa$ |
| $M_x$ (when $x < a$) | ........ | $= Px$ |
| $\Delta_{max}$ (at center) | ........ | $= \frac{Pa}{24EI}\left(3l^2 - 4a^2\right)$ |
| $\Delta_{max}$ (when $a = \frac{l}{3}$) | ........ | $= \frac{Pl^3}{28EI}$ |
| $\Delta_x$ (when $x < a$) | ........ | $= \frac{Px}{6EI}\left(3la - 3a^2 - x^2\right)$ |
| $\Delta_x$ (when $a < x < (l - a)$) | ........ | $= \frac{Pa}{6EI}\left(3lx - 3x^2 - a^2\right)$ |

# Example Problem

- Check Flexural Capacity

  Top Flange Fully Braced ($L_b$ = 0')

  $Z_{reqd} = M_u / \phi_b F_y$

  $= (347\text{k-ft})(12''/\text{ft}) / (0.9)(50\text{ ksi})$

  $= 92.6\text{ in}^3$

- ***Try W21x44*** ($Z = 95.4\text{ in}^3$; $I = 843\text{ in}^4$)

- Check Shear Capacity

  $\phi_v V_n$ = 195k *from tables*

  $> V_u = 41.7\text{k}$

$F_y$ = 50 ksi

# Table 3–2 (continued)
## W Shapes
### Selection by $Z_x$

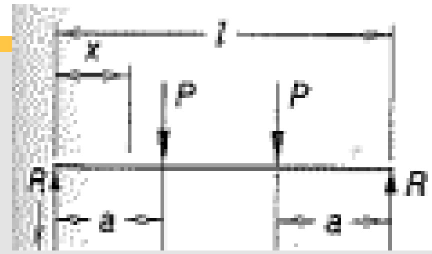
$Z_x$

| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF$ | | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| **W21×55** | **126** | **314** | **473** | **192** | **289** | **10.8** | **16.3** | **6.11** | **17.4** | **1140** | **156** | **234** |
| W14×74 | 126 | 314 | 473 | 196 | 294 | 5.34 | 8.03 | 8.76 | 31.0 | 795 | 128 | 191 |
| W18×60 | 123 | 307 | 461 | 189 | 284 | 9.64 | 14.5 | 5.93 | 18.2 | 984 | 151 | 227 |
| W12×79 | 119 | 297 | 446 | 187 | 281 | 3.77 | 5.67 | 10.8 | 39.9 | 662 | 116 | 175 |
| W14×68 | 115 | 287 | 431 | 180 | 270 | 5.20 | 7.81 | 8.69 | 29.3 | 722 | 117 | 175 |
| W10×88 | 113 | 282 | 424 | 172 | 259 | 2.63 | 3.95 | 9.29 | 51.1 | 534 | 131 | 197 |
| **W18×55** | **112** | **279** | **420** | **172** | **258** | **9.26** | **13.9** | **5.90** | **17.5** | **890** | **141** | **212** |
| **W21×50** | **110** | **274** | **413** | **165** | **248** | **12.2** | **18.3** | **4.59** | **13.6** | **984** | **158** | **237** |
| W12×72 | 108 | 269 | 405 | 170 | 256 | 3.72 | 5.59 | 10.7 | 37.4 | 597 | 105 | 158 |
| **W21×48**[f] | **107** | **265** | **398** | **162** | **244** | **9.78** | **14.7** | **6.09** | **16.6** | **959** | **144** | **217** |
| W16×57 | 105 | 262 | 394 | 161 | 242 | 7.98 | 12.0 | 5.65 | 18.3 | 758 | 141 | 212 |
| W14×61 | 102 | 254 | 383 | 161 | 242 | 4.96 | 7.46 | 8.65 | 27.5 | 640 | 104 | 156 |
| W18×50 | 101 | 252 | 379 | 155 | 233 | 8.69 | 13.1 | 5.83 | 17.0 | 800 | 128 | 192 |
| W10×77 | 97.6 | 244 | 366 | 150 | 225 | 2.59 | 3.90 | 9.18 | 45.2 | 455 | 112 | 169 |
| W12×65[f] | 96.8 | 237 | 356 | 154 | 231 | 3.60 | 5.41 | 11.9 | 35.1 | 533 | 94.5 | 142 |
| **W21×44** | **95.4** | **238** | **358** | **143** | **214** | **11.2** | **16.8** | **4.45** | **13.0** | **843** | **145** | **217** |
| W16×50 | 92.0 | 230 | 345 | 141 | 213 | 7.59 | 11.4 | 5.62 | 17.2 | 659 | 124 | 185 |
| W18×46 | 90.7 | 226 | 340 | 138 | 207 | 9.71 | 14.6 | 4.56 | 13.7 | 712 | 130 | 195 |
| W14×53 | 87.1 | 217 | 327 | 136 | 204 | 5.27 | 7.93 | 6.78 | 22.2 | 541 | 103 | 155 |
| W12×58 | 86.4 | 216 | 324 | 136 | 205 | 3.76 | 5.66 | 8.87 | 29.9 | 475 | 87.8 | 132 |
| W10×68 | 85.3 | 213 | 320 | 132 | 199 | 2.57 | 3.86 | 9.15 | 40.6 | 394 | 97.8 | 147 |
| W16×45 | 82.3 | 205 | 309 | 127 | 191 | 7.16 | 10.8 | 5.55 | 16.5 | 586 | 111 | 167 |


| Shape | $Z_x$ | $M_{px}/\Omega_b$ | $\phi_b M_{px}$ | $M_{rx}/\Omega_b$ | $\phi_b M_{rx}$ | $BF$ | | $L_p$ | $L_r$ | $I_x$ | $V_{nx}/\Omega_v$ | $\phi_v V_{nx}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | kip-ft | kip-ft | kip-ft | kip-ft | kips | kips | | | | kips | kips |
| | in.$^3$ | ASD | LRFD | ASD | LRFD | ASD | LRFD | ft | ft | in.$^4$ | ASD | LRFD |
| **W21×44** | **95.4** | **238** | **358** | **143** | **214** | **11.2** | **16.8** | **4.45** | **13.0** | **843** | **145** | **217** |
| W16×50 | 92.0 | 230 | 345 | 141 | 213 | 7.59 | 11.4 | 5.62 | 17.2 | 659 | 124 | 185 |
| W18×46 | 90.7 | 226 | 340 | 138 | 207 | 9.71 | 14.6 | 4.56 | 13.7 | 712 | 130 | 195 |
| W14×53 | 87.1 | 217 | 327 | 136 | 204 | 5.27 | 7.93 | 6.78 | 22.2 | 541 | 103 | 155 |
| W12×58 | 86.4 | 216 | 324 | 136 | 205 | 3.76 | 5.66 | 8.87 | 29.9 | 475 | 87.8 | 132 |
| W10×68 | 85.3 | 213 | 320 | 132 | 199 | 2.57 | 3.86 | 9.15 | 40.6 | 394 | 97.8 | 147 |


$\Omega_b$ = 1.67 | $\phi_b$ = 0.90
$\Omega_v$ = 1.50 | $\phi_v$ = 1.00

# Example Problem




- Check Serviceability
  - Live Load Deflection

    $\Delta_L = (P\ a\ /\ 24\ E\ I)\ (3l^2 - 4a^2)$

    $\Delta_{max}$ (at center) ......... $= \frac{Pa}{24\,EI}(3l^2 - 4a^2)$

    $= [(7.3k)(25'/3)(12''/ft)^3/(24)(29E3ksi)(843in^4)]\ [3(25')^2\text{-}4(25'/3)^2]$

    $= 0.29'' < L/360 = (25')(12''/ft)/360 = 0.83''$ *OK*
  - Dead Load + Live Load Deflection

    $\Delta_{D+L}$ = *ratio the deflections*

    $= (32.2k/7.3k)\ (0.29'')$

    $= 1.27'' \sim L/240 = (25')(12''/ft)/240 = 1.25''$ *close OK*

**Use W21x44 typical interior girder**